ADVANCED HIGHER MATHEMATICS

REVISION FOR UNIT 3

PRACTICE ASSESSMENT 1

Outcome 1

1. Given the vectors **a** = **i** + **k** and **b** = 3**i** – 2**j**, find the vector product **a** × **b**.

2. Obtain, in parametric form, the equation of the line which passes through the points A(1, 3, –2) and B(–1, 1, 3).

3. Find the equation of the plane which has normal vector **n** = 3**i** + 2**j** – 2**k** and passes through the point (2, –1, 1).

Outcome 2

The matrices A, B, C and D are given by

$$A = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix},\ B = \begin{pmatrix} 6 & 2 \\ 0 & 1 \end{pmatrix},\ C = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 1 & -1 \\ 0 & 2 & 2 \end{pmatrix}.$$

Find:
(a) the matrix $2A + B - C$
(b) the matrix AB
(c) the inverse matrix C^{-1}
(d) the determinant of matrix D.

Outcome 3

1. Find the first three terms of the MacLaurin series for $f(x) = e^{-2x}$.

2. The equation $x^3 - 2x - 3 = 0$ can be rewritten as $x = (2x+3)^{\frac{1}{3}}$.

   By using the simple iterative formula $x_{n+1} = (2x_n + 3)^{\frac{1}{3}}$ with $x_0 = 2$, find an approximation to the root of the equation which lies in the interval $1 < x < 3$, giving your answer correct to two decimal places.

Outcome 4

1. Find the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + \frac{1}{x}y = \sin x,$$

giving the solution in the form $y = f(x)$.

Outcome 5

1. Use the method of proof by induction to prove that, for all natural numbers $n \geq 1$,

$$\sum_{k=1}^{n} 6k^2 = n(n+1)(2n+1).$$

2. Use the Euclidean algorithm to find the greatest common divisor of 1696 and 1504.

PRACTICE ASSESSMENT 4

Outcome 1

1. Given the vectors **a** = **i** + 2**j** – **k** and **b** = 3**j** + 2**k**, find the vector product **a** × **b**.

2. Obtain, in parametric form, the equation of the line which passes through the points A(2, –1, 3) and B(1, 3, 5).

3. Find the equation of the plane which has normal vector **n** = **i** + 2**j** – **k** and passes through the point (4, 1, 2).

Outcome 2

The matrices A, B, C and D are given by

$$A = \begin{pmatrix} 1 & 5 \\ 3 & -1 \end{pmatrix},\ B = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix},\ C = \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 4 & -1 & -2 \end{pmatrix}.$$

Find: (a) the matrix $3A - 2B$ b) the matrix BC
(c). the inverse matrix C^{-1} (d) the determinant of matrix D.

Outcome 3

1. Find the first three terms of the MacLaurin series for $f(x) = \sin 2x$.

2. The equation $x^3 - 2x - 5 = 0$ can be rewritten as $x = (2x+5)^{\frac{1}{3}}$.

   By using the simple iterative formula $x_{n+1} = (2x_n + 5)^{\frac{1}{3}}$ with $x_0 = 2$, find an approximation to the root of the equation which lies in the interval $1 < x < 3$, giving your answer correct to two decimal places.

Outcome 4

1. Find the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + 2xy = xe^{-x^2},$$

giving the solution in the form $y = f(x)$.

Outcome 5

1. Use the method of proof by induction to prove that, for all natural numbers $n \geq 1$,

$$\sum_{k=1}^{n} (2k+1) = n(n+2).$$

2. Use the Euclidean algorithm to find the greatest common divisor of 2093 and 1679.

PRACTICE ASSESSMENT 3

Outcome 1

1. Given the vectors $\mathbf{a} = 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, find the vector product $\mathbf{a} \times \mathbf{b}$.

2. Obtain, in parametric form, the equation of the line which passes through the points A(1, −1, 2) and B(3, 2, 1).

3. Find the equation of the plane which has normal vector $\mathbf{n} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and passes through the point (−1, 3, 2).

Outcome 2

The matrices A, B, C and D are given by

$$A = \begin{pmatrix} 5 & 1 \\ 4 & 6 \end{pmatrix},\ B = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix},\ C = \begin{pmatrix} 4 & 2 \\ -5 & -3 \end{pmatrix} \text{ and } D = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 1 & -4 & 1 \end{pmatrix}.$$

Find:
(a) the matrix $A + B - 2C$
(b) the matrix AC
(c) the inverse matrix C^{-1}
(d) the determinant of matrix D.

Outcome 3

1. Find the first three terms of the MacLaurin series for $f(x) = \ln(1 + 2x)$.

2. The equation $x^3 - 2x - 7 = 0$ can be rewritten as $x = (2x + 7)^{\frac{1}{3}}$.

   By using the simple iterative formula $x_{n+1} = (2x_n + 7)^{\frac{1}{3}}$ with $x_0 = 2$, find an approximation to the root of the equation which lies in the interval $1 < x < 3$, giving your answer correct to two decimal places.

Outcome 4

1. Find the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + \frac{y}{x} = e^x,$$

giving the solution in the form $y = f(x)$.

Outcome 5

1. Use the method of proof by induction to prove that, for all natural numbers $n \geq 1$,

$$\sum_{k=1}^{n} 4k^3 = n^2(n+1)^2.$$

2. Use the Euclidean algorithm to find the greatest common divisor of 1147 and 851.